Problem #1

\( Y = \) Hardness  \\
\( X = \) Time  \\
Assume the normal error regression model.

1. Fit the model \( E(Y) = \beta_0 + \beta_1 X \).

2. Evaluate \( \sigma^2 \).

3. Evaluate \( \varepsilon \) when \( X = 10 \).

4. Test \( H_0 : \beta_1 = 1.85 \) Vs \( H_a : \beta_1 \neq 1.85 \).

5. Construct a 95% confidence interval for \( \beta_0 \).

6. Construct a 90% confidence interval for \( E(Y_b) \) when \( X_b = 60 \).

7. Find a 90% prediction interval for a new observation when time equals 60.

8. Comment on the residual plot.

9. Comment on the normal probability plot.

10. Conduct the correlation test for normality.

11. Conduct the Shapiro-Wilks test for normality.

12. Conduct the modified Levene test for constant error variance.

13. Conduct a Breusich-Pagen test for constant error variance.

14. Fit the model \( E(Y) = \beta_0 + \beta_1 X + \beta_2 X^2 \).

15. Test \( H_0 : \beta_2 = 0 \) Vs \( H_a : \beta_2 \neq 0 \).

17 Construct a 90% simultaneous confidence interval for \( \beta_0 \) and \( \beta_1 \) using the model in part 14.

18 Write down the SAS codes to generate the attached output. (Sorry)

**Problem #2**

Consider the model \( E(Y) = \beta_0 + \beta_1 X \).

Data: \[
\begin{array}{cc}
Y & X \\
5  & 1 \\
8  & 2 \\
11 & 1 \\
13 & 2 \\
\end{array}
\]

(a) Find the \( X \) matrix.

(b) Find the vector \( b \).

(c) Find the hat matrix.

(d) Find the vector \( \hat{Y} \).

(e) Find the vector \( \hat{\varepsilon} \).

(f) Transform the independent variable as \( Z = \frac{X - \bar{X}}{0.5} \).

Data: \[
\begin{array}{cc}
Y & Z \\
5  & 1 \\
8  & 1 \\
11 & 1 \\
13 & 1 \\
\end{array}
\]

(i) Find the new \( X \) matrix.

(ii) Find the new hat matrix.

(iii) Find the vector \( \hat{Y} \) (after transformation).

(iv) Find the vector \( \hat{\varepsilon} \) (after transformation).

(g) Comment on the hat matrix, \( \hat{Y} \), and \( \hat{\varepsilon} \) before and after transformation. What are the implications of this result?